

Simple Iterative Model for Adjusting Hazen-Williams Friction Coefficient for Drip Irrigation laterals.

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Abstract: Hazen-Williams equation is used widely by irrigation systems' designers due to its simplicity. However, Darcy-Weisbach equation is more accurate and reliable. The accuracy of the latter is due to its friction coefficient, which depends on both flow characteristics and pipe surface state. On the other hand, Hazen-Williams' coefficient (C) depends only on pipe substance and age. A comparative analysis of both models was performed through a simple iterative model. The analysis was based on the real state design procedure of drip laterals. More accurate values of coefficient C were suggested to be used in designing drip laterals. A straightforward equation was developed to compute C depending on emitter flowrate, emitter flow exponent, and pipe diameter. The results reveal that C ranges from 132 to 138 for drip laterals, while it was proved that using $C=150$ is reasonable for manifolds design.

Key words: Hazen-Williams, Darcy-Weisbach, Friction Coefficient, Churchill equation, Drip Irrigation laterals design.

INTRODUCTION

Designing a drip irrigation network is an operation in which pipelines diameters and lengths are determined and optimized for economic and efficient system operation. In practice, the dripper line diameter is predetermined by the manufacturing availability of lines with built in emitters, thus, most networks have 16mm outer diameter (OD) polyethylene (PE) dripper lines. In some cases, 13mm OD or 19mm OD dripper lines are used. The goal for designing dripper laterals is to determine its' maximum length to ensure acceptable uniformity over the subunit. On the other hand, designing manifolds deals more with specifying their diameters as their lengths are usually limited by network planning. However, sizing either type is mainly based on the friction losses limitation to the allowable amount. Friction losses are calculated by several methods. The most famous methods in irrigation design are Darcy-Weisbach (D-W), and Hazen-Williams (H-W).

(Allen, 1996) related D-W and H-W friction factors with Reynolds number, concluding the importance of adjusting H-W friction factor (C) with changing pipe velocity and diameter. (Moghazi, 1998) conducted some laboratory experiments to determine the proper values of H-W factor. He reported the values for the commonly used pipe sizes in trickle irrigation, and compared the percent of increase in friction due to fixing the value of C . Shayyaa and Sablani (1998) accomplished an artificial neural network to calculate the D-W friction factor, their model agrees very well with the Colebrook equation in the turbulent stage of the flow. (Valiantzas, 2005) compared the H-W and D-W friction factors, and developed a power function for this relation. (Martorano, 2006) achieved a comparative study between D-W and H-W, recommending the usage of D-W due to its precision. (Yildirim and Ozger, 2008) developed a Neuro-fuzzy approach in estimating H-W friction coefficient for small diameter polyethylene pipes. They proved that fixing H-W coefficient over all PE diameters might lead to considerable error in friction loss computation.

Friction losses calculation methods:

Friction losses calculation is most accurately performed by the Darcy-Weisbach equation (Eqn. 1).

$$h_f = f \frac{L v^2}{D 2g} \quad (1)$$

where h_f : pressure head loss due to friction (L), f : friction factor, L : pipe length (L), D : pipe diameter (L), v : water velocity (LT^{-1}), and g : acceleration of gravity (LT^{-2}). The friction factor f depends on Reynolds number

(R_N) and the relative roughness of the pipe (R_R). f could be evaluated if R_N and R_R are known by graphical or analytical means, through Moody diagram (Moody, 1944) or Churchill equation (Churchill, 1977) respectively.

$$R_N = \frac{\rho \cdot v \cdot D}{\mu} \tag{2}$$

$$R_R = \frac{e}{D} \tag{3}$$

where ρ : water density (ML^{-3}), v : velocity (LT^{-1}), D : pipe inner diameter (L), μ : viscosity ($ML^{-1}T^{-1}$), and e : mean roughness height along pipe inner surface. Although there are several equations used to evaluate the friction factor, but the Churchill equation, Eqn.(4), is the only one valid for all types of flow, laminar, turbulent, and even transient. For that reason, Churchill equation is used in this work.

$$f = 8 \left[\left(\frac{8}{R_N} \right)^{12} + \left(\left\{ -2.457 \ln \left[\left(\frac{7}{R_N} \right)^{0.9} + 0.27 R_R \right] \right\}^{16} + \left\{ \frac{37530}{R_N} \right\}^{16} \right)^{-\frac{3}{2}} \right]^{1/12} \tag{4}$$

As noticed, Churchill equation requires R_R to be known, while it is not easy to be measured on all pipe lifetime. Moreover, e varies due to the quality of water used, quantity of sediments in it, age of pipe, pipe wall material and finishing, and some other minor causes. Accordingly, most of the designers use Hazen-Williams equation, Equ. (5), which depend only on a single factor called capacity factor (C) which relies on the pipe material and age, where it varies from 80 to 150 from very rough to very smooth pipes. (Williams and Hazen, 1933).

$$h_f = K \frac{L}{D^{-4.875}} \left(\frac{Q}{C} \right)^{1.852} \tag{5}$$

Where K : units parameter = 1.21E10 when substituting D in [mm], Q in [l/s], and L in [m].

In this study, a simple iterative model were developed to establish the relationship between Hazen Williams C and Darcy-Wiesbach e/D , in order to find the amount of rectification needed to C to make the usage of H-W equation more accurate.

Model Development:

In designing a drip irrigation subunit, the allowable friction loss is adjusted to minimize variations between emitters in the subunit not to exceed 10% of the emitter’s nominal discharge (ASABE, 2008). This amount of discharge tolerance is converted to pressure through the emitter equation, Eqn.(6) , so as it is affected by the emitter flow exponent x , as shown in Eqn.(7).

$$q = kh^x \tag{6}$$

$$\frac{dq}{q} = x \frac{dh}{h} \tag{7}$$

where k : emitter flow parameter [L^3T^{-1}], h : pressure head [L]. As noticed by Eqn.(7), for an emitter with $x=0.5$; 10% variation in discharge is equivalent to 20% variation in pressure head. However, this amount is usually distributed between laterals and manifolds as 55% and 45% respectively. For example, if the emitter’s equation is $q=1.265h^{0.5}$, so as $q=4l/s$ @ $h=10m$, then the allowable pressure variation is 20% of 10m, i.e. 2m. Therefore, the allowable losses in lateral lines and manifolds are 1.1m, and 0.9m respectively. Most of the designers allow the emitter to operate minimally on its nominal discharge, so the far-end emitter in the subunit will operate at 10m head, and the near-end emitter will operate at 12m head, as illustrated in Fig. 1.

Starting from the far-end emitter, with the minimum desired discharge, moving against water direction, friction losses are calculated for each segment depending on the passing flow, which increases in the opposite-flow direction as illustrated in Fig 2.

On each segment, total friction losses are summated and compared to the allowable friction loss in laterals. The suitable lateral line length is calculated as in Eqn(8).

$$\text{if } \sum_{i=1}^{i=n} h_{f_i} \geq h_{f_{\text{allowable in line}}} \quad \text{Then : } L_{\text{Suitable}} = (n - 1) \times \ell \quad (8)$$

Consequently, the allowable line length varies according to the friction equation. D-W method is considered as the proper reference for allowable line length. Hence, the related H-W factor C could be established. However, establishing C cannot be done directly, as the friction loss of the line is computed from summation of segments losses. So, an iterative method should be used. For each relative roughness value, the reference allowable length (L_{all}) was found using D-W method then, L_{all} was established again using H-W method with C values in the range 70 to 160. The equivalent C estimate is the value that leads to the closer allowable line length to the D-W method. For example, if D-W gives an L_{all} equal to 52m, while at H-W method with different values of C with their corresponding L_{all} values. An interval found with boundaries of L_{all} 50m and 60m at $C=120$, and 125 respectively. The closer C value should be 120. But for more accurate calculations, C should be evaluated with the relative interval method, Eqn.(9), in this case $C=121$.

$$C = C_1 + \left((L_{DW} - L_1) \times \frac{(C_2 - C_1)}{(L_2 - L_1)} \right) \quad (9)$$

where L_{DW} is the L_{all} value established from D-W method, subscripts $_1$ and $_2$ indicate the first and second boundaries of the interval where L_{DW} laid inside, as mentioned above. To establish the relationships between C and the related variables; a spreadsheet model was developed; whereas the effect of each variable was cleared. The variables and their values are shown in Table 1. A detailed flowchart of the developed model is illustrated in the appendix.

RESULTS AND DISCUSSION

Relative roughness (e/D) versus capacity factor (C):

A simulation run was performed for eight values of pipeline diameter as mentioned in Table 1. The first three diameters were considered as lateral lines, while the rest five diameters were considered manifolds. For each case, the nine relative roughness values were applied and the corresponding capacity value was computed. The entire simulation was executed for the two mentioned nominal discharges of the emitter. The simulation results are summarized in Fig 3.

As noticed in Fig. 3, the C value tends to increase as the roughness decreases (R8 is the roughest pipe and R0 is almost very smooth), this coincides with the typically expected trend. However, in H-W equation, the C value is set to be 150 for very smooth pipes, and the shown trends approaches this value in most cases. The shown diameters were classified into two groups; laterals group and manifolds group. In the laterals group C value starts from about 70 (at R8) to about 130 (at R0). The 4L/h discharge emitter C values are almost 10 units less than their corresponding values of the 2 L/h emitter. On the other hand, the manifold group is uniform, starting from below 70 units to unite in the standard value of 150 (at R0). This results lead us to conclude that using $C=150$ with H-W equation in designing drip lateral lines is incorrect, while using C in the range 130 to 135 is more reliable to get accurate results. While using $C=150$ for designing manifolds is acceptable for very smooth pipes like PVC, and for the early ages of the pipe before roughness increases due to sediments and chemicals. However, if the designed network is supposed to suffer lack of maintenance and management, then the C factor should lose 10% of its value to be about 135 for manifolds as a factor of safety.

In Fig. 4, it can be noticed that emitter discharge effect is very small, while the roughness series are divided into two groups; smooth group and rough group. Smooth group contains only one curve (R0), while the rest of relative roughness levels are set in the other group. The capacity factor tends to increase with the pipe diameter (D) in the smooth group, while it contrarily decreases with D in the rough group. This could be contributed that for rough pipes, as D increases the mean roughness e also increases (as the relative roughness is fixed). Therefore, the equivalent capacity factor moves toward the roughness direction (less C value). For the smooth group, as D increases the flow resistance decreases (with no wall roughness), therefore the equivalent capacity factor moves against the roughness direction (more C value).

Line Length As Affected By Pipe Roughness and Emitter Exponent:

Determining line length is a main goal while planning or designing drip networks. However, many designers consider the line length as an experience-mentioned property. Actually, line length varies widely with the amount of flow, emitters' characteristics, and pipe roughness. Fig. 5 shows a sample illustration of a 13 mm lateral line with different combinations of emitter discharge, emitter flow exponent, and pipeline roughness. It could be noticed that line length is inversely proportional to emitter discharge, emitter exponent, and to pipe

roughness as well. It is noticeable that the line length resulted from using H-W with $C=150$ is longer than that of D-W at R0. This is because the overestimate of $C=150$ to express the lateral lines as mentioned before.

However, designing lateral lines using H-W eqn. with $C=150$ leads to longer lines, and therefore to drop in performance and increase in friction losses. The increment in friction losses was calculated by the model as follows:

$$FIP = \frac{h_{f_{H.W}} - h_{f_{D.W}}}{h_{f_{D.W}}} \tag{10}$$

where FIP : friction increment percent due to using H.W instead of D.W, (%), h_f : friction head loss in line (L), H.W: calculated using Hazen-Williams method, D.W: calculated using Darcy-Weisbach method. a brief study of lateral diameters is illustrated in Table 2. In this table, it is noticed that FIP varies from 6% to 18.4%, this value increases with the increment of x in most cases. It, however, increases with the decrease of emitter flow rate, and decreases as the diameter increases. These results outshoot the importance of the proper assignment of the C value as mentioned before.

It could also be noticed in Fig. 5 that the effect of emitter flow-exponent (x) is very impressive, it may result to more than 200% increase in the line length (40m @ $x=1.00$, and 125m @ $x=0.05$ in the 2 L/h chart). This leads to the importance of using pressure-compensating emitters or at least as low x values as possible. The H-W coefficient C is directly affected by x , this could be attributed as the inconsistency of the flow increases by the increase of x , so that the friction inside the pipe increases which leads to a higher C value as shown in Table 3.

C Values Deviation From No-Exits Laterals:

The current model deals only with lateral lines with emitters installed, while all of the mentioned literature deal with no-exits lines. Fig. 6 illustrates the resulted ranges of the current model, compared to the results of two of the no-exits works. It can be noticed that the current model's range of 16mm pipeline lies within the range of the two other models, while it has some bias below average in the 19mm pipeline, and above average in the 13mm pipeline. This bias could be attributed to effect of the emitters' existence, where the line has a continuous decreasing flowrate.

It could also be noticed from Fig. 6 that the proposed model is less spreading in value than the other models, so an average value of C could be taken safely with minimum error. Table 4 shows the standard deviation comparison of each research work. Equation (11) shows a simple relation to obtain C as a function of x , D , and q .

$$C = 129.81 + 0.314D + q - 7.556x \tag{11}$$

The adjusted correlation coefficient of the equation is $r_{adj}=0.8948$. and the standard error is, $SE:1.2579$.

Summary and Conclusions:

Owing to the importance of Hazen-Williams equation in designing drip irrigation networks, the friction factor of it was adjusted to give the same results as the accurate Darcy-Weisbach equation. To achieve this goal, a simple iterative model was developed, a comparative analysis was made, and a simple equation was presented to compute the appropriate C values. The results of this paper agrees with previous works with some bias due to difference in analysis methods, and because these works dealt with a no-exits lateral line while this paper dealt with real state laterals with emitters installed on it. The results showed that using the proper values reduces the friction loss error by up to 18.4%, and hence, lead to safer and more reliable drip irrigation networks.

Table 1: Variables used in the model and its values.

Variable	Values										
	From		To		Step		Count				
x	0.05		1.00		0.05		20				
C	70		160		5		19				
(e/D)	Label	R8	R7	R6	R5	R4	R3	R2	R1	R0	9
	Value	0.225	0.149	0.075	0.056	0.037	0.0187	0.0075	0.0037	0.000075	
$q_{(nominal)}$	(2 l/h), (4 l/h)										2
D	(13mm), (16mm), (19mm), (50mm), (63mm), (75mm), (90mm), (110mm)										8
<i>Number of alternatives</i>										54'720	

Nomenclature

Symbol	Meaning
ρ	water density [ML ⁻³]
μ	viscosity [ML ⁻¹ T ⁻¹]
C	Hazen-Williams capacity factor
D	pipe diameter [L]
D	Darcy-Weisbach
$-W$	
e	mean roughness height along pipe inner surface [L]
f	Darcy-Weisbach friction factor
f(...)	function of ...
F	friction increment percent
IP	
g	acceleration of gravity [LT ⁻²]
h	emitter pressure head [L]
h_a	allowable head loss [L]
h_e	emitter head [L]
h_f	pressure head loss due to friction [L]
h	minimum allowable head in the subunit [L]
h_{min}	

Symbol	Meaning
H	Hazen-Williams
$-W$	
i, j, k	Counters (in flowchart)
K	units parameter
k	emitter flow parameter [L ³ T ⁻¹]
l	pipe/segment length [L]
L	Total pipe length [L]
L	Allowable length [L]
Q	pipe discharge [L ³ T ⁻¹]
q	emitter discharge [L ³ T ⁻¹]
q_{em}	
q_s	segment discharge [L ³ T ⁻¹]
R	Reynolds number
R	relative roughness of the pipe
v	water velocity [LT ⁻¹]
x	emitter flow exponent
\Rightarrow	.. should be increased by ...

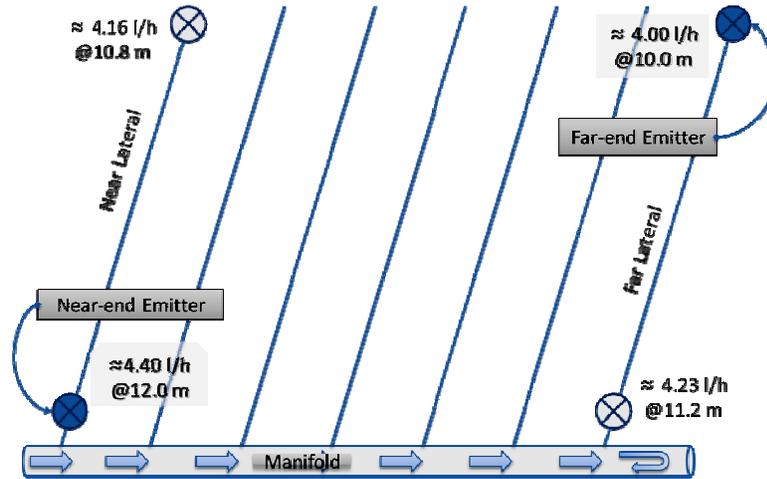


Fig. 1: Drip irrigation subunit, showing extreme emitters discharge and operating pressure

Table 2: Maximum length of lateral lines with different diameters calculated using both D-W formula (RR=R0), and H-W formula (C=150). Friction Increase Percent (FIP) and Equivalent C at R0 are shown.

	2 L/h					4 L/h				
	x	L_{DW-R0}	L_{HW-150}	FIP	$Equv.C$	x	L_{DW-R0}	L_{HW-150}	FIP	$Equv.C$
13 mm	0.05	118.00	124.50	10.14%	138.00	0.05	76.00	79.50	8.48%	140.00
	0.10	91.50	97.50	12.07%	136.00	0.10	59.00	62.00	9.36%	138.33
	0.50	50.50	55.50	18.22%	130.00	0.50	32.50	35.00	14.16%	135.00
	0.75	44.00	48.00	16.73%	132.50	0.75	28.00	30.50	16.43%	132.50
	1.00	39.50	43.00	16.31%	130.00	1.00	25.00	27.50	18.40%	130.00
16 mm	x	L_{DW-R0}	L_{HW-150}	FIP	$Equv.C$	x	L_{DW-R0}	L_{HW-150}	FIP	$Equv.C$
	0.05	166.50	175.00	9.40%	139.38	0.05	107.00	111.50	7.74%	141.00
	0.10	129.00	137.00	11.41%	136.67	0.10	83.00	87.00	8.87%	138.75
	0.50	71.50	77.50	15.44%	132.50	0.50	46.00	49.50	14.00%	135.00
	0.75	61.50	67.50	17.95%	130.00	0.75	39.50	42.50	13.98%	132.50
1.00	55.50	61.00	18.24%	130.00	1.00	35.50	38.50	15.55%	132.50	
19 mm	x	L_{DW-R0}	L_{HW-150}	FIP	$Equv.C$	x	L_{DW-R0}	L_{HW-150}	FIP	$Equv.C$
	0.05	262.00	273.50	8.08%	140.42	0.05	168.50	174.00	6.01%	142.50
	0.10	203.00	214.00	9.97%	138.00	0.10	130.50	136.50	8.4%	140.00
	0.50	112.50	121.50	14.72%	133.33	0.50	72.50	77.50	12.69%	136.25
	0.75	97.00	105.50	16.13%	132.00	0.75	62.50	67.00	13.25%	135.00
1.00	87.50	95.00	15.77%	132.00	1.00	56.00	60.50	14.79%	133.33	

Table 3: Hazen-Williams capacity variable (C) calculated by comparing allowable lateral lengths to Darcy-Wiesbach values for several emitter flow-exponents.

Emitter flow exponent	Darcy-Wiesbach Equation	Lateral line* allowable length when calculating friction losses using: Hazen-Williams Equation with C value=											Proper C Value
		80	85	90	95	100	105	110	130	140	150	160	
0.05	57.5	52.5	54.5	57.0	59.0	61.0	63.0	64.5	72.0	76.0	79.5	82.5	91.25
0.10	45.5	41.0	43.0	44.5	46.0	47.5	49.0	50.5	56.5	59.5	62.0	64.5	93.33
0.20	36.0	32.0	33.5	34.5	36.0	37.0	38.5	39.5	44.0	46.5	48.5	50.5	95.00
0.30	31.5	28.0	29.0	30.0	31.0	32.0	33.5	34.5	38.5	40.0	42.0	44.0	97.50
0.40	28.5	25.0	26.0	27.0	28.0	29.0	30.0	31.0	34.5	36.5	38.0	39.5	97.50
0.50	26.5	23.0	24.0	25.0	26.0	27.0	27.5	28.5	32.0	33.5	35.0	36.5	97.50
0.60	25.0	21.5	22.5	23.5	24.5	25.0	26.0	27.0	30.0	31.5	33.0	34.5	100.00
0.70	23.5	20.5	21.5	22.0	23.0	24.0	24.5	25.5	28.5	29.5	31.0	32.5	97.50
0.80	22.5	19.5	20.5	21.0	22.0	22.5	23.5	24.0	27.0	28.5	29.5	31.0	100.00
0.90	22.0	18.5	19.5	20.5	21.0	21.5	22.5	23.0	26.0	27.0	28.5	29.5	102.50
1.00	21.0	18.0	19.0	19.5	20.0	21.0	21.5	22.5	25.0	26.0	27.5	28.5	100.00
* This case: D=13 mm, e=0.5 mm, q=4 L/h											Average C for this case: 97.46 ≈ 100		

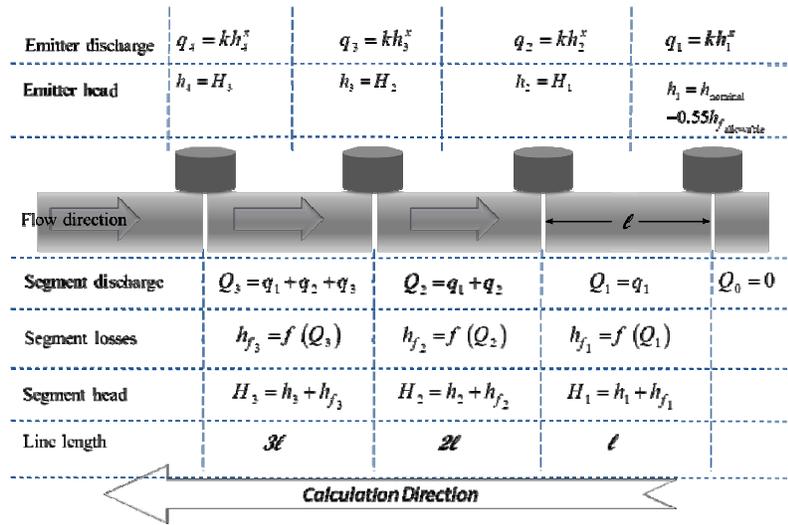
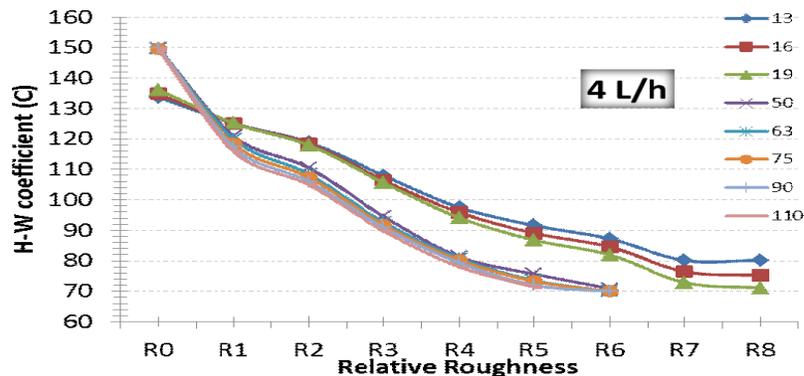


Fig. 2: Segment by segment calculation of drip lateral line.

Table 4: Average and Standard Deviation values comparison of H-W C factor.

q_{em}	D	Proposed		Moghazi (1998)		Yildirim and Ozger (2008)	
L/h	Mm	Avg	SD	Avg	SD	Avg	SD
2	13	132.3	2.140	129.7	4.031	129.8	3.073
2	16	132.6	2.527	136.3	5.418	137.0	5.656
2	19	133.7	2.535	144.1	4.703	144.3	4.464
4	13	133.8	2.880	129.7	4.031	129.8	3.073
4	16	134.8	2.674	136.3	5.418	137.0	5.656
4	19	136.1	2.534	144.1	4.703	144.3	4.464



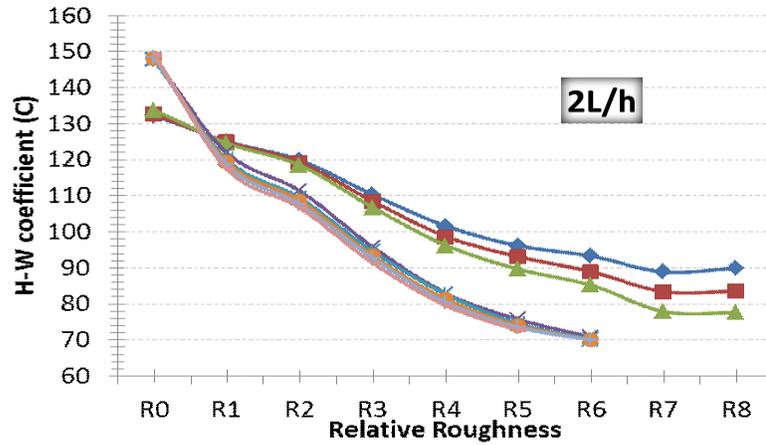


Fig. 3: Relative roughness versus capacity factor, compared at several pipeline diameters and two emitter nominal discharges.

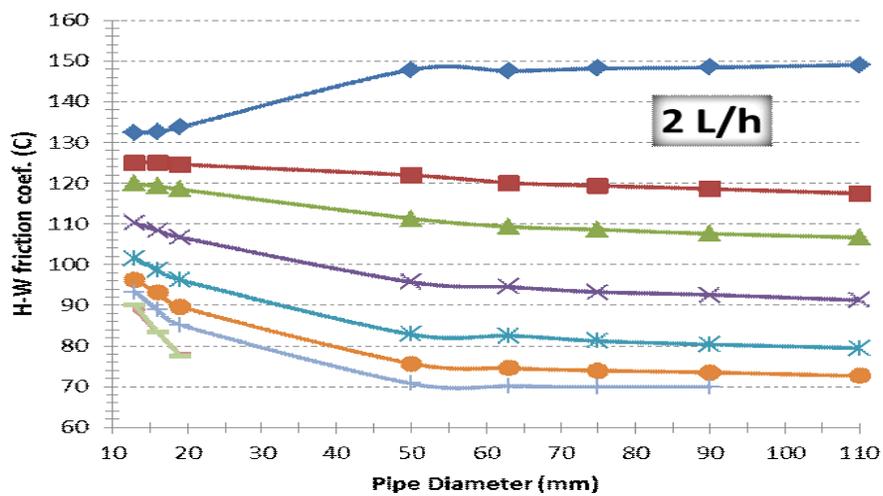
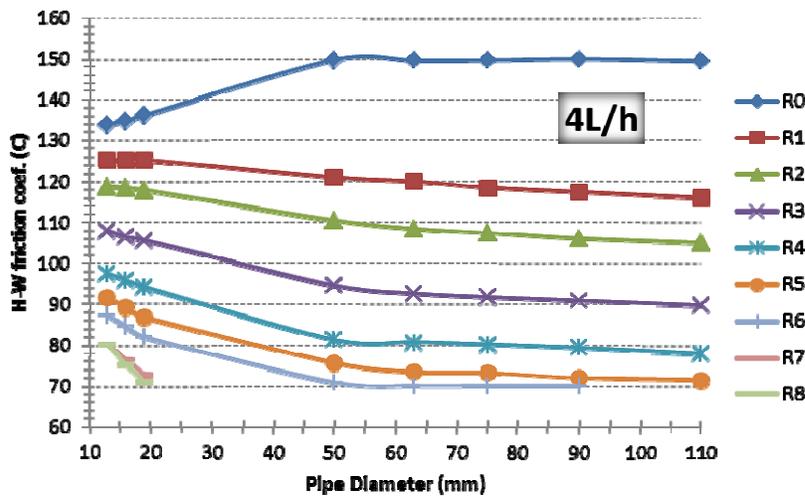


Fig. 4: Pipeline diameters effect on capacity factor, at several relative roughness values and two emitter nominal Discharges.

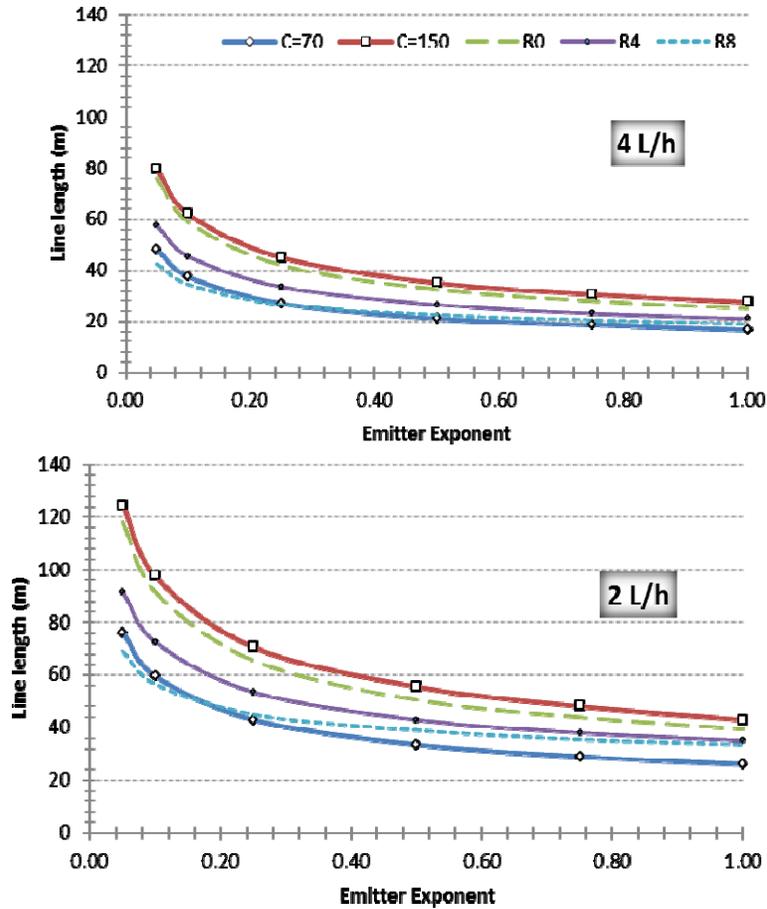


Fig. 5: Maximum allowable line length of a 13mm lateral line with different emitter flow exponents, emitter discharges of 2 and 4 L/h, for several pipe roughness.

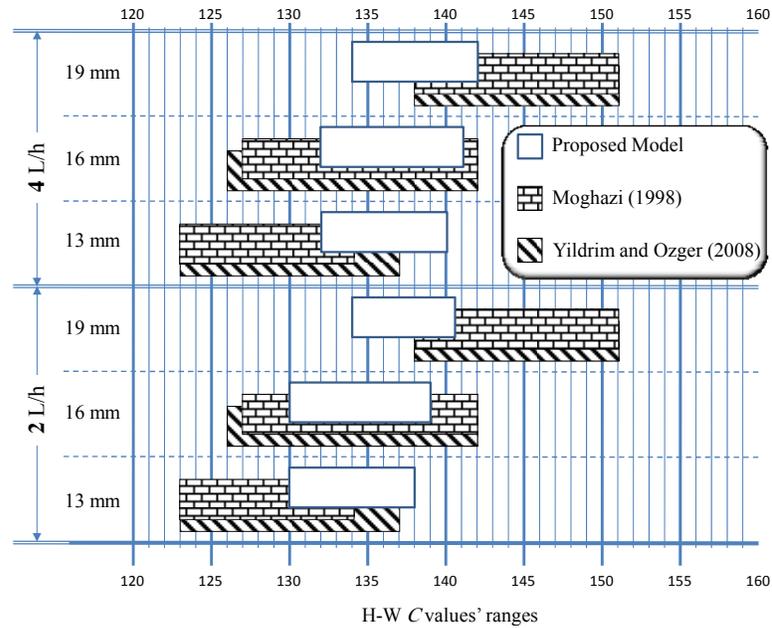


Fig. 6: Comparison chart between C ranges resulted from current model and two other models.

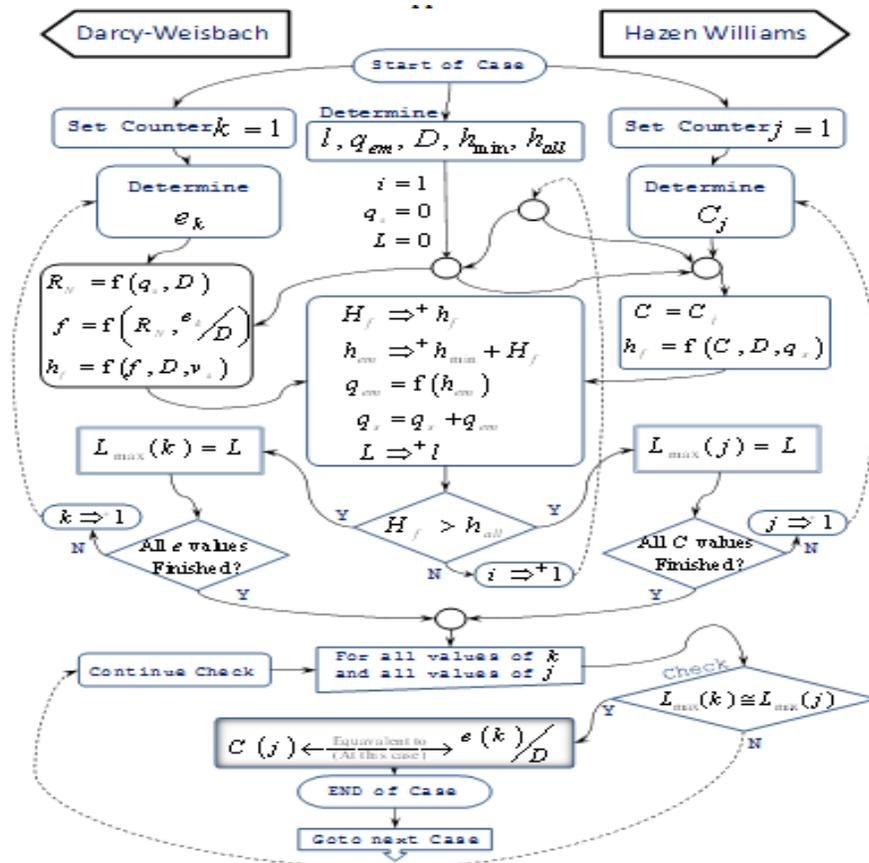


Fig. 7: Flowchart representing the model procedure for determining the equivalent friction factors of H-W and D-W. (Symbols are defined in the nomenclature).

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