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# Simple statistical equivalents of Penman–Monteith formula's parameters in the absence of non-basic climatic factors

**Mohammad N. ElNesr · Abdulrahman A. Alazba**

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**Abstract** The Penman–Monteith (PM) method is the most recommended method for estimating reference evapotranspiration ($ET_o$). The PM equation requires several parameters to be available, either measured or computed. Some of these parameters are conventionally calculated by some slightly sophisticated formulas, especially for handy calculations. This paper aimed to derive some simpler statistical equivalents to these formulas. Simplifications were performed to the formulas of the saturation vapor pressure $e_o[T]$, slope of vapor pressure ($\Delta$), atmospheric pressure ($P$), the psychrometric constant ($\gamma$), wind speed correction, the long-wave radiation, $R_{nl}$; the sunset hour angle, $\omega_s$; and the extraterrestrial radiation, $R_a$. For the first five parameters, the parameter-independent factor was analyzed for its extremes, then fitted by interpolation to a simpler equivalent formula. The last three parameters were fitted to simpler form through data from the FAO-CLIMWAT database. Each of the simplified formulas was compared to the conventional one; some correlation indices were applied to validate the new formulas. The $ET_o$ was calculated for all stations in the CLIMWAT database by both simplified and conventional formulas. All the correlation results were excellent, with a minimum correlation coefficient of 0.9966. The simplified formulas were proven to be equivalent in performance, with almost no loss in accuracy but simpler in form and faster in execution in the online database applications.

M. N. ElNesr (✉) · A. A. Alazba
Alamoudi Water Chair, King Saud University,
K.S.A. Riyadh, POB 2460, Riyadh 11451, Saudi Arabia
e-mail: drNesr@gmail.com

A. A. Alazba
e-mail: Alazba@ksu.edu.sa

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## Introduction

Estimating reference evapotranspiration ($ET_o$) is the basic step toward the calculation of water requirements for irrigated lands. Several models were developed to predict $ET_o$ from basic or complex meteorological parameters. The most recommended model to predict it is the Food and Agricultural Organization (FAO) Penman–Monteith (PM) procedure presented in Allen et al. (1998). This method is commonly known as the FAO-56 method.

The FAO-56 Penman–Monteith equation, determining evapotranspiration from standard grass reference, is expressed as follows (Allen et al. 1998):

$$ET_o = \frac{0.408\Delta(R_n - G) + \frac{900}{T_a+273}\gamma U_2(e_s - e_a)}{\Delta + \gamma(1 + 0.34U_2)} \quad (1)$$

where $ET_o$ is the reference evapotranspiration (mm day$^{-1}$); $R_n$ is the net radiation at the crop surface (MJ m$^{-2}$ day$^{-1}$); $G$ is the soil heat flux density (MJ m$^{-2}$ day$^{-1}$); $T_a$ the mean daily air temperature (°C); $U_2$ the wind speed at 2-m height (m s$^{-1}$); ($e_s - e_a$) the vapor pressure deficit (kPa); $e_s$ the saturation vapor pressure (kPa); $e_a$ the actual vapor pressure (kPa); $\Delta$ the slope vapor pressure curve (kPa °C$^{-1}$); and $\gamma$ is the psychrometric constant (kPa °C$^{-1}$).

As noticed, the PM formula consists of eight parameters which can be measured directly or indirectly using specific devices in some meteorological stations (MTSs). The basic records of any MTS are temperature, relative humidity, and wind speed. In many of the agricultural MTSs, actual sunshine hours and vapor pressure are recorded too.

However, only one of the eight parameters is always measured by any MTS, i.e., the temperature. The remaining seven parameters are either reformed after measurement ($U_2$, adjusted for sensor altitude), estimated directly from one or more measured parameters ($\gamma$, depends on the air pressure and station altitude), or indirectly estimated from measured parameters ($R_n$, estimated from the bright sunshine hours and other measured parameters). If someone dealt with an MTS that does not measure one or more of the mentioned parameters, he has to estimate it using some formulas, which causes some confusion and difficulty. The complexity of calculations increases as each of these parameters could be expressed by a variety of units, thus resulting in some significant errors (Valiantzas 2006).

To eliminate the muddle and its corresponding errors, several investigators suggest or perform a simplification of the formulas or formula procedure. Linacre (1992, 1993) developed some simplified formulas for the original Penman equation to estimate open water evaporation and crop evapotranspiration. Alazba (2001) reported that the accuracy of PM equation changed meaningfully on adapting the equation parameters. Nandagiri and Kovoor (2005) performed a sensitivity measure of alternative formulas used to evaluate $ET_o$ parameters. They concluded the need of revising the recommended calculation methods of $ET_o$ parameters. Valiantzas (2006) derived some simplified versions of the original Penman equation in order to calculate $ET_o$ using basic climate station's data. The basic assumption of these formulas is setting the station's altitude to zero. In one of his suggested formulas, wind speed parameter was removed, substituting by a common value of 2 m/s. Jabloun and Sahli (2008) evaluated the FAO-56 method using limited climatic data. They studied $ET_o$ calculated with the absence of some PM parameters compared to $ET_o$ with all data measured. They reported that using some alternative methods to compute missing parameters leads to a small statistical error. The aim of this paper was to derive some simple and accurate formulas to evaluate missing climatic factors with fewer number of calculating steps and easier procedure.

## Mathematical procedure

### Simplification of vapor pressure formulas

The saturation vapor pressure at specific temperature is the basic component for computing the actual and saturation vapor pressure, the vapor pressure deficit, and the slope of vapor pressure. The original relationship between $T$ and $e_o$ was derived by Tetens (1930), as in Eq. 13 in the Appendix. As seen in the formula, we have to substitute $T$ two times, apply multiplication, addition, and rising to the natural base power, as well as using parenthesis. This may cause confusion and lead to some errors unless perfectly preprogrammed as a function in the spreadsheet software. This formula can be fitted to the modified power model in the form: $e_o[T] = a \times b^T$, where $a$ and $b$ are fitting parameters; $T$ appears only once and the equation includes only two simple operations. For the temperature range −10°C to 60°C, the equivalent form is as follows:

$$e_o[T] = 0.783 \times 1.057^T. \quad (2)$$

This relation is very well correlated and in good agreement with Eq. 13. The coefficient of determination, ($r^2$) = 0.998, and the standard error (SE) = 0.134. This form can be substituted to Eq. 12 to compute $e_s$ and in Eq. 14 to find $e_a$.

As mentioned, the slope of vapor pressure curve Δ is affected by $e_o[T_a]$, as developed by Murray (1967) (Eq. 16 in the Appendix). For the temperature range −10°C to 60°C, the original form was fitted to the modified power model, and a truly simpler relationship was derived, which has $r^2$= 0.998 and SE=0.0126.

$$\Delta = 0.047 \times 1.057^{T_a}. \quad (3)$$

### Simplification of formulas based on station altitude ($z$)

The psychrometric constant $\gamma$ is expressed as in Eq. 17 in the Appendix. $\gamma$ is a function of the atmospheric pressure $P$ and the latent heat flux $\lambda$. The atmospheric pressure is expressed as in Eq. 18 in the Appendix, which is a sort of power equation. This formula has a simpler equivalent which is in a simple linear form (Eq. 4), with $r^2$=0.9978 and SE=0.51

$$P = 101.06 - 0.0105z. \quad (4)$$

The latent heat flux $\lambda$, Eq. 19, can be approximated to 2.45 as reported by Harrison (1963) for $T_a$=20°C. Substituting Eq. 4 in Eq. 17 with the value of $\lambda$=2.45, the psychrometric constant can be calculated as follows:

$$\gamma = 0.0672 - 7.56 \times 10^{-6}z. \quad (5)$$

For further simplification, for altitude ranges from 0 to 1,000 m, $\gamma$ can be approximated to 0.064.

$$\gamma \approx 0.064 \rightarrow \text{for } 0z \leq 1,000. \quad (6)$$

If the wind speed sensor is not at 2-m height as required for the PM equation, the measured wind speed value has to be modified by Eq. 7 as a simpler form of Eq. 21, which was derived using power fit module, with $r^2$=0.9994 and SE= 0.0004, so that the equivalent wind speed formula is:

$$U_2 = 1.09\ U_z\ (z_h - 0.3)^{-0.17} \quad (7)$$

where $U_z$ is the wind speed ($ms^{-1}$) measured at height $z_h$ (m).

**Table 1** Statistical indices used to evaluate the reliability of the simplified formulas

| Index name | Formula | Measures |
|---|---|---|
| Coefficient of correlation | $CC = \frac{\frac{1}{n}\sum_{i=1}^{n}(\Theta_i-\overline{\Theta})(\Psi_i-\overline{\Psi})}{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(\Theta_i-\overline{\Theta})^2}\sqrt{\frac{1}{n}\sum_{i=1}^{n}(\Psi_i-\overline{\Psi})^2}}$ | Relationship performance |
| Nash–Sutcliffe coefficient | $NE = \sum_{i=1}^{n}(\Theta_i-\Psi_i)^2 \Big/ \sum_{i=1}^{n}(\Theta_i-\overline{\Theta})^2$ | Relative estimation of model performance |
| Root mean square error | $RE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\Theta_i-\Psi_i)^2}$ | Residual variance |
| Mean absolute error | $ME = \frac{1}{n}\sum_{i=1}^{n}\lvert\Theta_i-\Psi_i\rvert$ | Prediction closeness |

$\Theta_i$ {*theta*}: value calculated by conventional equation, $\Psi_i$; {*psi*}: value calculated by simplified formula; *i*: counter; *n*: count of variable cases; $\overline{\Theta}$: average of $\Theta_i$ values; $\overline{\Psi}$: average of $\Psi_i$ values

## Simplification of radiation formulas

The original formula of the long-wave radiation, $R_{nl}$ (Eq. 25), is in a complex form. Several simplifications were applied to each term of it; hence, a simpler form was derived as follows:

$$R_{nl} = 0.0128(100 + T_x + T_n)(2.43 - \sqrt{e_s})\left(3.86\,\frac{R_s}{R_{so}} - 1\right). \quad (8)$$

In case of missing $R_s$ data

$$R_{nl} = 0.0123(100 + T_x + T_n)(2.43 - \sqrt{e_s}) \times \left(5.14\, c\,\sqrt{T_x - T_n} - 1\right) \quad (9)$$

where $c$ = 0.19 and 0.16 for coastal and non-coastal regions, respectively.

Although these forms are not that simple, compared to the original equation, they contain no power-four factors, no exponential numbers, and even no need to transfer temperatures to Kelvin as all temperatures here are in Celsius.

Usually, we need to calculate the extraterrestrial radiation $R_a$ when $R_s$ data are missing. $R_a$ is normally calculated by Eq. 27 in the Appendix, which is a function of many variables. The latitude data $\varphi$ are critical and cannot be eliminated or approximated, but the solar declination variable $\delta$ can accept some modifications. By analyzing possible values of $\delta$ for all range of Julian day values $J$ (Eq. 30), it was found that $-0.41<\delta<+0.41$ ($\pm 23°_{max}^{Deg}$). However, $\delta$ is almost a small angle. For small angles, both angle sine and tangent can be replaced by the radian value of the angle itself, while the cosine of small angles is almost equal to unity. The sunset hour angle, $\omega_s$, formula (Eq. 31) is equivalent to a simpler formula ($r^2$=0.994, SE=0.027):

$$\omega_s = \frac{\pi}{2} + 1.16\,\delta\,\tan\varphi. \quad (10)$$

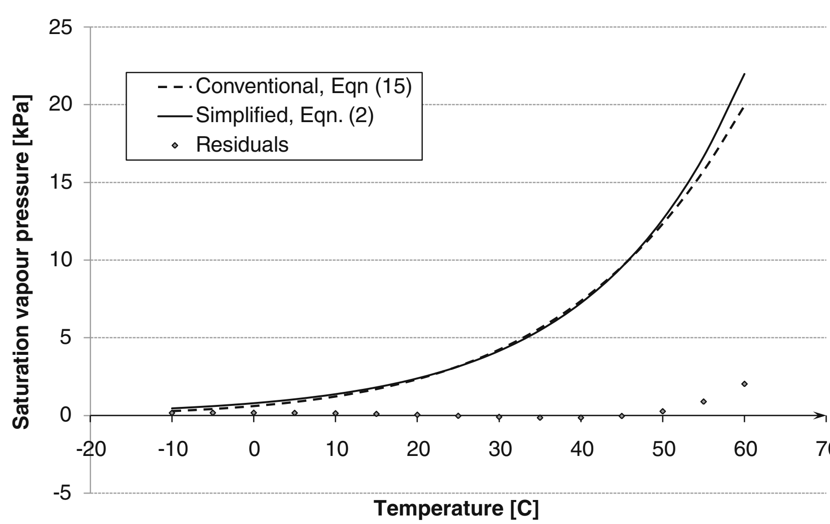



**Fig. 1** Comparison between conventional vs. simplified formulas to calculate saturated vapor pressure from temperature

**Table 2** Statistical indices of the simplified parameters

| Parameter Symbol | Conventional equation no. | Simplified equation | | | Statistical indices | | | | Figure no. |
|---|---|---|---|---|---|---|---|---|---|
| | | No. | $r^2$ | SE | CC | ME | RE | NC | |
| $e_o[T]$ | Eq. 13 | Eq. 2 | 0.9986 | 0.1340 | 0.9983 | 0.3062 | 0.5899 | 0.9913 | 1 |
| $\Delta$ | | | 0.9978 | 0.0126 | 0.9990 | 0.0118 | 0.0134 | 0.9976 | 2 |
| $P$ | | | 0.9979 | 0.5100 | 0.9995 | 0.2915 | 0.3403 | 0.9985 | 3 |
| $U_2$ | Eq. 20 | Eq. 7 | 0.9974 | 0.0004 | 0.9988 | 0.0035 | 0.0042 | 0.9975 | 4 |
| $R_{nl}$ | Eq. 25 | Eq. 8 | – | – | 0.9992 | 0.0705 | 0.0928 | 0.9962 | 7 |
| $\omega_s$ | Eq. 31 | Eq. 10 | 0.9940 | 0.0270 | 0.9989 | 0.0184 | 0.0265 | 0.9978 | 5 |
| $R_a$ | Eq. 27 | Eq. 11 | – | – | 0.9973 | 0.8137 | 1.0398 | 0.9945 | 6 |

From trigonometric equivalence, rearranging and substituting in Eq. 27, $R_a$ is approximated to:

$$R_a \cong 36\ (\ \delta\ \omega_s\ \sin\varphi + \sin\omega_s\ \cos\varphi). \qquad (11)$$

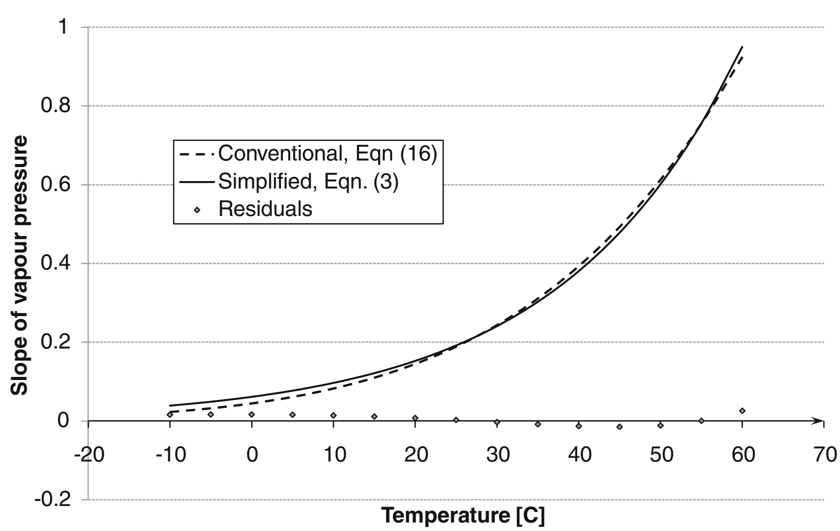
## Verification of simplified formulas

The simplified formulas divide into two groups: The first group includes formulas that contain only one independent variable, i.e., $e_o[T]$, $\Delta$, $P$, $\gamma$, and $U_2$. The second group includes multi-independent formulas, i.e., $R_{nl}$, $\omega_s$, and $R_a$. The verification of the first group requires comparing the simplified version vs. the conventional one using the independent variable through its range of validity. For example, $e_o[T]$ and $\Delta$ depend on temperature (independent variable); thus, the range of validity to compare with is −10°C to 60°C. On the other hand, verification of the second group requires real climatic data for different stations because of the infinite number of possible combinations of the independent variables. As suggested by Kim and Kim (2008), the reliability of the simplified formulas was measured using four statistical indices. Two of them measure the prediction performance—the coefficient of correlation (CC) and the Nash–Sutcliffe coefficient (NC)—and two indices measure the scattering—root mean square error (RE) and mean absolute error (ME). Formulas of these indices were shown in Table 1.

### The single-variable group verification

The saturation vapor pressure function $e_o[T]$ was computed using conventional and simplified equations in the temperature range −10°C to 60°C (Fig. 1). It is clear that both equations match very well and the residuals are almost zero, except above 50°C where the simplified equation overestimates the conventional equation slightly. The relationship performance is extremely high, with CC=0.9986; so is the relative estimation of model performance that was measured by the Nash–Sutcliffe coefficient (NE=0.9913). The scattering indices also gave good values, where RE=0.589 and ME=

**Fig. 2** Slope of vapor pressure curve as affected by temperature, computed using conventional and simplified formulas



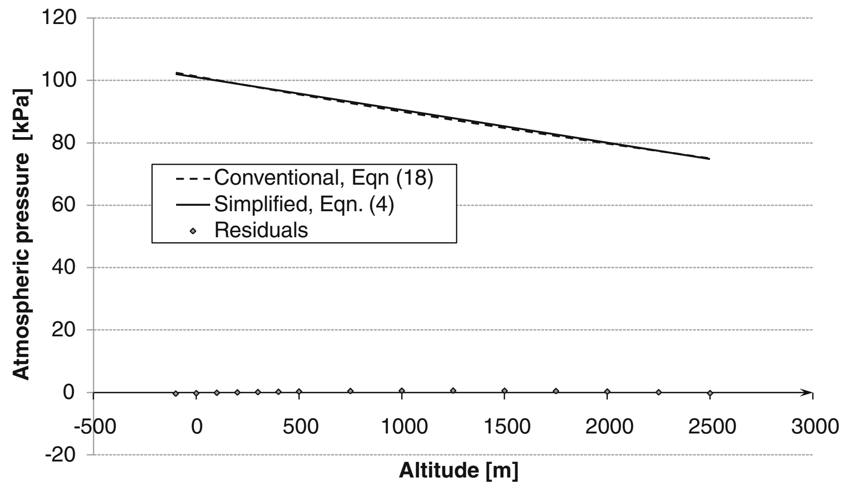



**Fig. 3** Atmospheric pressure as affected by station's altitude, computed using conventional and simplified formulas

0.306. The results of the statistical indices of all the simplified equations are summarized in Table 2. Similar, but better, results were found for the slope of vapor pressure curve Δ (Fig. 2), with CC=0.9990, NC=0.9976, RE=0.0134, and ME=0.0118, which reflect an excellent representation of the simplified equation to the conventional one. The equivalent form of the atmospheric pressure (Eq. 4) is compared to the conventional form (Eq. 18), showing excellent match (Fig. 3), with CC=0.9995, NC=0.9985, RE=0.3403, and ME=0.2915. This excellent match was tested for altitudes from −100 to 2,500 m, which covers the topmost of the world's irrigated lands. Nonetheless, in Fig. 4, the wind speed correction factor was computed for sensor height range 0.5–4 m by both the simplified and the original formulas (Eqs. 7 and 21, respectively). The figure shows noticeable coincidence between both formulas, except for sensor height below 0.8 m, which is not practical to be used. The statistical indices of the wind speed correction factor show an excellent representation of the simplified equation, as shown in Table 2.

## Multivariable parameter verification

The rest of the variables, including ET itself, belong to the second group that requires real data to verify by. We chose to verify these variables using actual data from the FAO CLIMWAT, as suggested by Valiantzas (2006). This database consists of 3,207 stations in 142 countries, where most of the climatic condition variabilities were covered. CLIMWAT provides long-term monthly mean values of nine climatic parameters. These parameters are: mean daily maximum temperature (°C), mean daily minimum temperature (°C), mean relative humidity (%), mean wind speed (km/day), mean sunshine hours per day, mean solar radiation (MJ$\text{m}^{-2}$ $\text{day}^{-1}$), monthly rainfall (mm/month), monthly effective rainfall (mm/month), and reference evapotranspiration calcu-

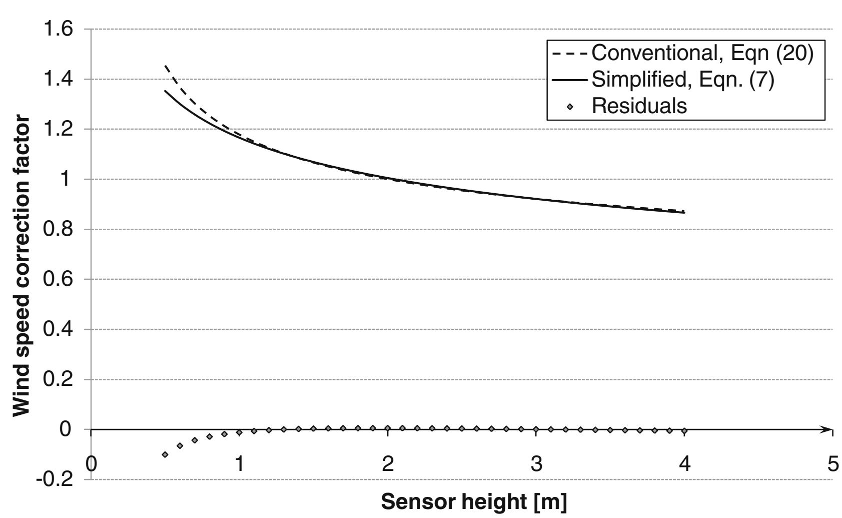



**Fig. 4** Wind speed correction factor computed using conventional and simplified formulas

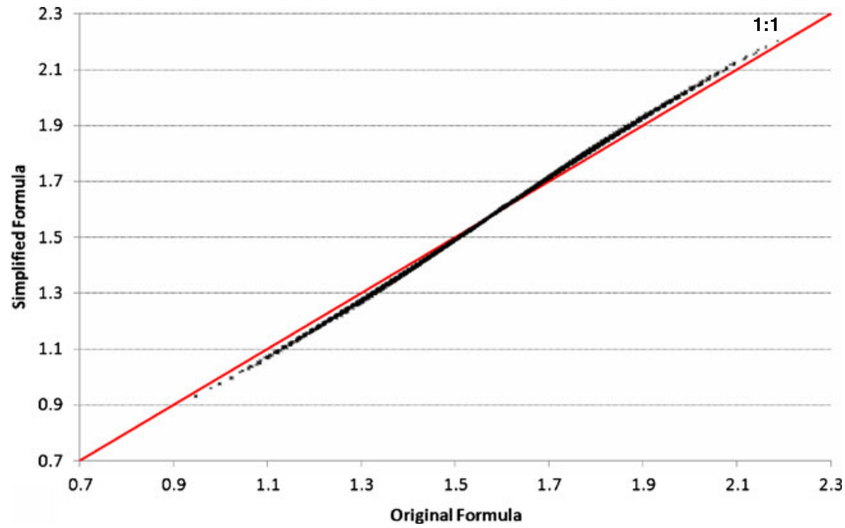



**Fig. 5** Comparison of values of the sunset hour angle, $\omega_s$, calculated by original and simplified formulas

lated with the Penman–Monteith method (mm/day). All of the mentioned parameters are of daily basis logged on averaged monthly basis.

The simplified form of the sunset hour angle, $\omega_s$ (Eq. 10), compared to the conventional formula (Eq. 31) shows a very high correlation (Fig. 5), with CC=0.9998 and RE=0.0152 confirming the validity of the simplified equation. On the other hand, the simplified form of the extraterrestrial radiation, $R_a$ (Eq. 11), is not that coincident with the original form (Eq. 27) as the CC=0.9853 and the RE=1.2627. Figure 6 shows this relation. Although it appears that there are some bias between the original and simplified forms of the equation, but it can be considered acceptable due to the small relative error to the predicted value (maximum of 8%), as well as the small contribution of the $R_a$ factor itself to the calculated ET, as will be shown later. With similar correlation results, the long-wave radiation, $R_{nl}$, conventional form (Eq. 25), was compared to its simplified forms (Eqs. 8 and 9) for measured or missing $R_s$ data, respectively. Figure 7 shows these correlations on the 1:1 perfect line. In case that is measured, the correlation of $R_{nl}$ appears to be very good, with CC=0.9725 and RE=0.2025 (Fig. 7a). Better correlation values were achieved for $R_{nl}$ with missing $R_s$ (Fig. 7b). The correlation values were CC=0.9927 and RE=0.2050 ($R_s$ is calculated using Eq. 24). Correlation results appear better for formulas that depend on calculated $R_s$ because of the vanishing of $R_a$ from the calculations when dividing Eq. 24 by Eq. 26, which minimizes the cumulative error.

## Resultant evapotranspiration comparison

To ensure the overall accuracy of the simplified equations, we made a comparison between the $ET_o$ calculated using

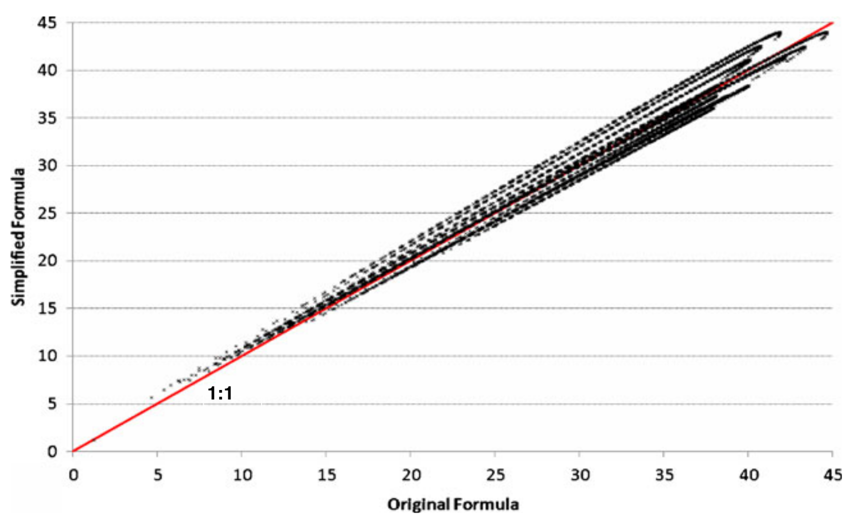



**Fig. 6** Comparison of values of the extraterrestrial radiation, $R_a$, calculated by original and simplified formulas

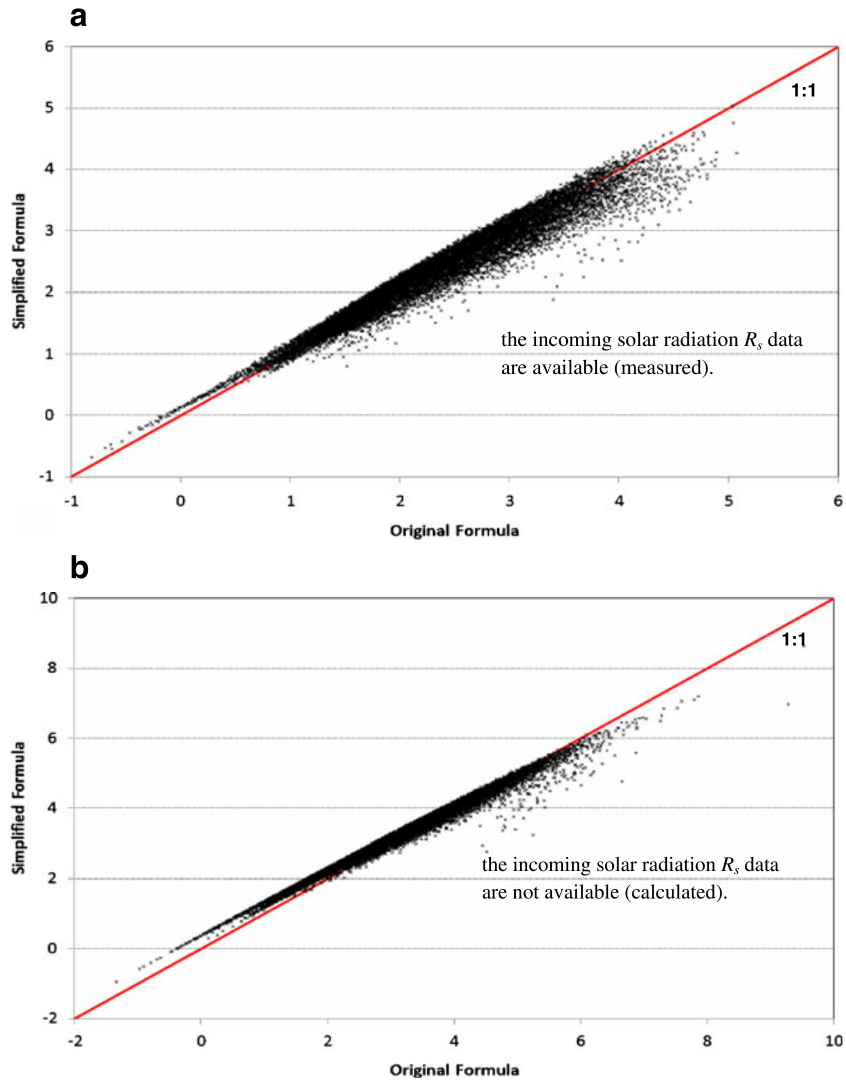



**Fig. 7** **a** Comparison of values of the long-wave radiation, $R_{nl}$, calculated by original and simplified formulas. **b** Comparison of values of the long-wave radiation, $R_{nl}$, calculated by original and simplified formulas

original formulas, in the Appendix, vs. the ET calculated using the derived statistical equivalent formulas. All comparisons were performed on the same dataset of the FAO-CLIMWAT. The conventional method was applied using Eq. 1 with the parameters $\Delta$ and $U_2$ calculated using Eqs. 16 and 21, respectively. Parameter $\gamma$ is calculated by substituting Eqs. 18 and 19 in Eq. 17. The vapor pressure deficit ($e_s - e_a$) was calculated by Eqs. 12, 13, and 15. Finally, parameter $R_n$ is computed by subtracting Eq. 25 from Eq. 23. Although $R_s$ data exist in the dataset, two comparisons were made: one with measured data and the other assuming that $R_s$ is missing. In case of missing $R_s$, the extraterrestrial radiation $R_a$ was computed first using Eqs. 27, 28, 29, 30, and 31 then substituted in Eq. 24 to find $R_s$. The simplified method was applied using Eq. 1, with the parameters $\Delta$ and $U_2$ calculated using Eqs. 3 and 7, respectively. The vapor pressure deficit was calculated based on Eq. 2. $\gamma$ was calculated using Eq. 5. $R_{nl}$ is computed by Eq. 8 or 9 in case of measured or missing $R_s$ data, respectively. $R_{ns}$ is computed using Eq. 22 by substituting the value of $R_s$ if exists; otherwise, $R_a$ was computed using Eq. 11 by substituting from Eqs. 29, 30, and 31.

When $R_s$ is measured, the $ET_o$ calculated using the simplified formulas ($ET_{os}$) correlated very well with the $ET_o$ calculated using conventional formulas ($ET_{oc}$, Fig. 8a), with CC=0.9994 and RE=0.0785. The figure shows how tight the scatter line is to the 1:1 line, which confirms the perfect representation of the simplified formulas to the conventional ones. With similar results, $ET_{os}$ correlated very well with $ET_{oc}$ when $R_s$ is missing (and calculated, Fig. 8b), with CC=0.9966 and RE= 0.1543, reflecting very good agreement with conventional equations as well.

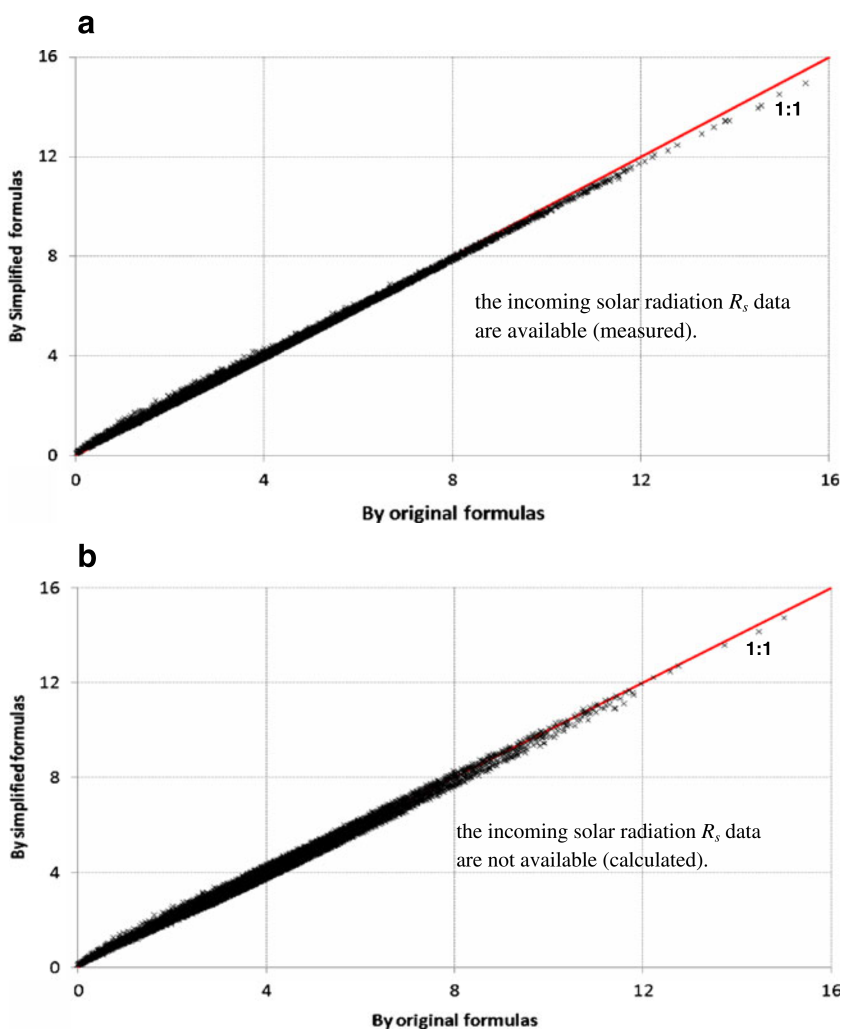



**Fig. 8** **a** Comparison of values of the reference evapotranspiration, $ET_o$, calculated using original and simplified formulas. **b** Comparison of values of the reference evapotranspiration, $ET_o$, calculated using original and simplified formulas

## Summary and conclusion

New statistical equivalents of eight parameters needed for $ET_o$ calculation using the FAO Penman–Monteith method were derived to simplify the calculation formulas' structure. The resultant equations are simpler, easier to calculate using handy calculators, and faster in computing using computers in database applications. All of the derived formulas were tested for correlation with conventional formulas, where the results show excellent correlation with CC > 0.997. Finally, the simplified formulas were used to calculate $ET_o$ for 142 countries worldwide in the FAO CLIMWAT climatic database. These countries almost have all variabilities in climate, which ensures perfect representation of the simplified formulas to the real world. Using this database, $ET_o$ values were calculated using both the simplified and conventional formulas. The correlation results were great, with a correlation coefficient of 0.9994 and root mean square error of 0.0785 mm/day, which confirms the reliability of the simplified formulas.

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## Appendix: The original equations

Vapor pressure deficit ($e_s - e_a$)

The vapor pressure deficit is the difference between the saturation and actual vapor pressure. It is an accurate indicator of the actual evaporative capacity of the air, sometimes called saturation deficit (Allen et al. 1998). The saturation vapor pressure ($e_s$) is the corresponding pressure at which equilibrium is reached between the water molecules escaping and returning to a water reservoir, and air cannot store any extra water molecules (Allen et al. 2005):

$$e_s = 0.5(e_o[T_n] + e_o[T_x]) \tag{12}$$

where $e_o[T]$ is the saturation vapor pressure at air temperature $T$, $T_n$ is the minimum dry bulb air temperature (°C), and $T_x$ the maximum dry bulb air temperature (°C). The relationship between $T$ and $e_o$ was derived by Tetens (1930) as follows:

$$e_o[T] = 0.611 \exp\left(\frac{17.27\ T}{T + 237.3}\right). \tag{13}$$

The actual vapor pressure $e_a$ is the vapor pressure exerted by the water in the air, which is obtained from Eq. 14 if the relative humidity and temperature data are present.

$$e_a = 0.005(\mathrm{RH}_x e_o[T_n] + \mathrm{RH}_n e_o[T_x]) \tag{14}$$

where $\mathrm{RH}_x$ is the maximum relative humidity (%) and $\mathrm{RH}_n$ is the minimum relative humidity (%).

In the absence of $\mathrm{RH}_x$ and $\mathrm{RH}_n$ and the presence of the average relative humidity ($\mathrm{RH}_a$), the following equation is used:

$$e_a = 0.005\mathrm{RH}_a(e_o[T_n] + e_o[T_x]). \tag{15}$$

### Slope of vapor pressure curve Δ

Saturation vapor pressure curve shows how $e_o[T]$ is affected by temperature. However, this relation is expressed in exponential or power forms as in Eqs. 13 and 2, respectively. The slope of the saturation vapor pressure curve (Δ) is an important parameter in describing vaporization and computed by the following equation as in Murray (1967):

$$\Delta = \frac{4098 \times e_o[T_a]}{(T_a + 237.3)^2} \tag{16}$$

where $T_a$ is the average dry bulb temperature, better if measured, or can be computed as an average of $T_x$ and $T_n$.

### The psychrometric constant $\gamma$

$$\gamma = 0.00163\frac{P}{\lambda} \tag{17}$$

where $P$ is the atmospheric pressure (kPa) and $\lambda$ is the latent heat flux (MJkg$^{-1}$).

Atmospheric pressure is expressed as in Eq. 18 (Burman et al. 1987):

$$P = 101.3\left(\frac{293 - 0.0065z}{293}\right)^{5.26} \tag{18}$$

where $z$ is the altitude (m).

The latent heat $\lambda$ depends on the average temperature, as in Eq. 19:

$$\lambda = 2.5 - 0.00236\ T_a. \tag{19}$$

### Wind speed

The wind speed in Eq. 1 should be evaluated at 2-m height above ground level. In cases where wind speed sensor was laid at a different height, the equivalent wind speed, $U_2$, could be calculated as follows:

$$U_2 = U_z\left(\frac{\ln\left(\frac{(2-d)}{z_0}\right)}{\ln\left(\frac{(z_h-d)}{z_0}\right)}\right) \tag{20}$$

where $U_z$ is the wind speed (ms$^{-1}$) measured at height $z_h$, $z_h$ is the actual height of wind speed measurement device (m), $d$ is the zero plane displacement of wind profile (m), and $z_o$ is the roughness parameter for momentum (m). However, for a standard reference crop with crop height 0.12 m, $U_2$ could be obtained from Eq. 21 (FAO 1990).

$$U_2 = U_z\left(\frac{4.87}{\ln(67.8\ z_h - 5.42)}\right) \tag{21}$$

### Net radiation $R_n$ (MJm$^{-2}$day$^{-1}$)

The net radiation refers to the balance between the energy absorbed, reflected, and emitted by the earth's surface. It is calculated as the difference between short-wave incoming radiation, $R_{ns}$, and long-wave outgoing radiation, $R_{nl}$, where $R_n = R_{ns} - R_{nl}$. The short-wave radiation is calculated by Eq. 23 (FAO 1990).

$$R_{ns} = (1 - \alpha)R_s \tag{22}$$

where $\alpha$ is the coefficient of canopy reflection (albedo) and $R_s$ is the incoming solar radiation (MJm$^{-2}$day$^{-1}$). But for grass surface, the albedo, $\alpha$=0.23; hence:

$$R_{ns} = 0.77R_s. \tag{23}$$

Allen et al. (1998) reported a validated formula to calculate $R_s$ from air temperature difference:

$$R_s = c\,R_a\sqrt{T_x - T_n} \tag{24}$$

where $R_a$ is the extraterrestrial radiation (MJm$^{-2}$day$^{-1}$) and $c$ is an adjustment coefficient, which is 0.19 for coastal regions and 0.16 for non-coastal regions, however, to reduce the number of calculation steps. On the other hand, the long-wave radiation is computed by Eq. 25:

$$R_{\mathrm{nl}} = 2.45 \times 10^{-9}\left(1.35\frac{R_{\mathrm{s}}}{R_{\mathrm{so}}} - 0.35\right) \times \left(0.34 - 0.14\sqrt{e_s}\right)\left(T_{\mathrm{x}_k}^4 + T_{\mathrm{n}_k}^4\right) \quad (25)$$

where the suffix $k$ indicates that $T_{\mathrm{x}}$ and $T_{\mathrm{n}}$ are in Kelvin ($T_{\mathrm{Kelvin}}$=$T_{\mathrm{Celsius}}$+273.15). $R_{\mathrm{so}}$ is the short-wave radiation for clear skies (MJm$^{-2}$day$^{-1}$) calculated from Eq. 26:

$$R_{so} = 0.75R_a. \quad (26)$$

*Extraterrestrial radiation $R_a$*

If measured $R_{\mathrm{s}}$ values are available, then there is no need to compute $R_{\mathrm{a}}$ as it vanishes from $R_{\mathrm{nl}}$ calculation. In the case of missing $R_{\mathrm{s}}$ measurements, $R_a$ should be calculated as follows (FAO 1990):

$$R_{\mathrm{a}} = 37.6\, d_{\mathrm{r}}(\omega_s \sin\varphi \sin\delta + \sin\omega_{\mathrm{s}} \cos\varphi \cos\delta) \quad (27)$$

where $d_{\mathrm{r}}$ is the relative distance from Earth to Sun, $\delta$ is the solar declination (rad), $\varphi$ is the latitude (rad), and $\omega_{\mathrm{s}}$ is the sunset hour angle (rad).

$$d_r = 1 + 0.033\ \cos(0.0172\, J) \quad (28)$$

where $J$ is the Julian day, which can be calculated by Craig's (1984) formula (Eq. 29):

$$J = \mathrm{int}\left(\frac{275}{9}M + D - 30\right) \quad (29)$$

if $M$>2 And Leap Year → Subtract 1
if $M$>2 And Not Leap Year → Subtract 2

where $M$ is the month of the year and $D$ is the day of the month. Julian day ranges from 1 to 366 (in leap year). The corresponding $d_{\mathrm{r}}$ values range from 0.97 to 1.03 and can be safely approximated to 1.0. The solar declination angle $\delta$ is computed according to Duffie and Beckman (1980) as follows:

$$\delta = 0.409\sin(0.0172J - 1.39). \quad (30)$$

The sunset hour angle $\omega_{\mathrm{s}}$ is computed as follows according to Duffie and Beckman (1980):

$$\omega_{\mathrm{s}} = \arccos(-\tan\varphi \tan\delta). \quad (31)$$

*Soil heat flux G*

For daily measurements of evapotranspiration, soil heat flux is the amount of daily stored energy in soil per unit area (MJm$^{-2}$day$^{-1}$). For average moist soil, $G$ could be calculated as in Wright and Jensen (1972) as:

$$G = 0.38(T_{a_n} - T_{\mathrm{a}_{n-1}}) \quad (32)$$

where $T_{\mathrm{a}_n}$ is the current day average temperature (°C) and $T_{\mathrm{a}_{n-1}}$ is the previous day's average temperature (°C). On the other hand Allen et al. (2005) stated that for daily periods, $G$ beneath a fully vegetated grass or alfalfa is relatively small compared to $R_{\mathrm{n}}$. Therefore, it can be approximated as $G = 0$, while according to the same reference, $G$ may be calculated as a ratio of $R_{\mathrm{n}}$. Hence, $G$ could be represented as:

$$G = 0.2\ R_n \quad (33)$$

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